CHAPTER 7

VECTORS AND FORCES

LEARNING OBJECTIVES

Upon completion of this chapter, you should be able to do the following:

1. Add, subtract, and determine the components of vectors.
2. Solve problems involving forces.
3. Solve problems involving translational and rotational equilibrium.

INTRODUCTION

The last chapter in this course deals with vectors and forces. Any study of vectors and forces requires a knowledge of trigonometry.

VECTORS

A *scalar quantity* is one that has magnitude only; that is, 10 watts, 4 miles, 17 acres, and 28.2 pounds per square inch. A *vector quantity* is one that has both magnitude and direction; that is, 6 miles due north, 250 knots at 30°, and 400 miles per hour to the west. Scalar quantities are represented by italicized letters. Vector quantities are represented by placing arrows over the italicized letters, for instance, $\vec{A}$; and the magnitude of the vector quantity is represented by the italicized letter of the vector quantity. Vectors are geometrically represented by arrows. The arrowhead represents the terminal end of a vector and indicates the vector's direction. The other end of the vector is called the initial end. The magnitude of the vector is the vector's length.

Two vectors are said to be equal if they are of the same length, are parallel, and point in the same direction. In figure 7-1, view A, $\vec{A} = \vec{B}$.

If two vectors have the same length, are parallel, but point in opposite directions, they are said to be negatives of each other. In figure 7-1, view B, $\vec{C} = -\vec{D}$ and $\vec{D} = -\vec{C}$.

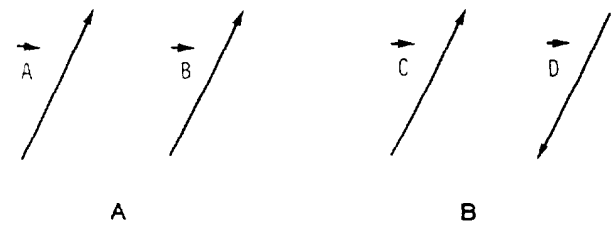

Figure 7-1.—Equal and negative vectors.

Hence, a vector can be moved from one position to another without being changed if its direction and magnitude are kept unchanged.

VECTOR ADDITION

The general rule for adding vectors is illustrated in figure 7-2. To add $\vec{B}$ to $\vec{A}$, shift $\vec{B}$ until its initial end coincides with the terminal end of $\vec{A}$. In its new position, $\vec{B}$ will be parallel, the same length, and in the same direction it was in the old position of $\vec{B}$. To find the vector sum of $\vec{A} + \vec{B}$, draw a vector, $\vec{R}$, with its initial end at the initial end of $\vec{A}$ and its terminal end at the terminal end of $\vec{B}$. Hence, $\vec{R}$ is called the *resultant vector* of $\vec{A}$ and $\vec{B}$, which is written

$$\vec{R} = \vec{A} + \vec{B}$$

If we reverse the process to add $\vec{A}$ to $\vec{B}$, we would move $\vec{A}$ until its initial end coincides with the terminal end of $\vec{B}$ so that

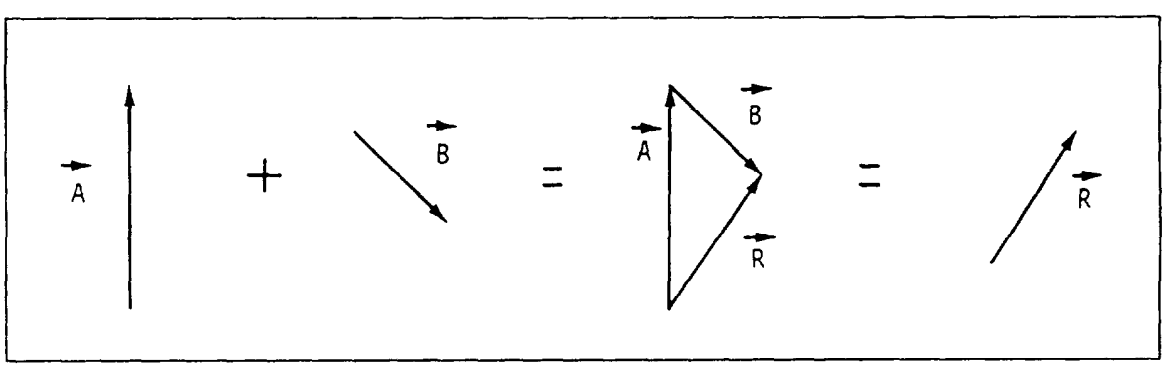

Figure 7-2.—Addition of $\vec{B}$ to $\vec{A}$.

$\vec{R}$ is also the resultant vector of $\vec{B} + \vec{A}$. (See fig. 7-3.) Thus

$$\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

To find the resultant vector of any number of vectors, place the initial end of each vector to the terminal end of the previous vector. Be sure the new vector position is parallel, the same length, and in the same direction as the old position. Draw a vector $\vec{R}$, from the initial end of the first vector to the terminal end of the last vector so that $\vec{R}$ is the resultant vector. Figure 7-4 shows how four vectors are added together. We recognize again that the order in which the vectors are added does not affect the result. (See fig. 7-5.) Thus,

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{D} + \vec{C} + \vec{B} + \vec{A}$$

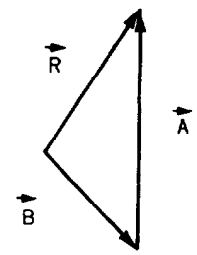

Figure 7-3.—Addition of $\vec{A}$ to $\vec{B}$.

VECTOR SUBTRACTION

We can also subtract one vector from another vector. Refer to figure 7-6. To subtract $\vec{B}$ from $\vec{A}$, we need to determine

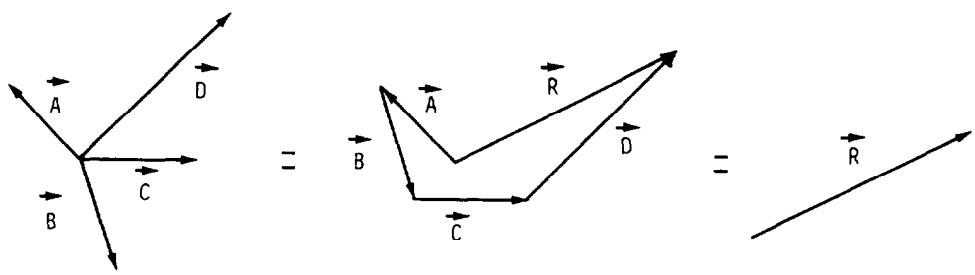

Figure 7-4.—Addition of $\vec{A} + \vec{B} + \vec{C} + \vec{D}$.

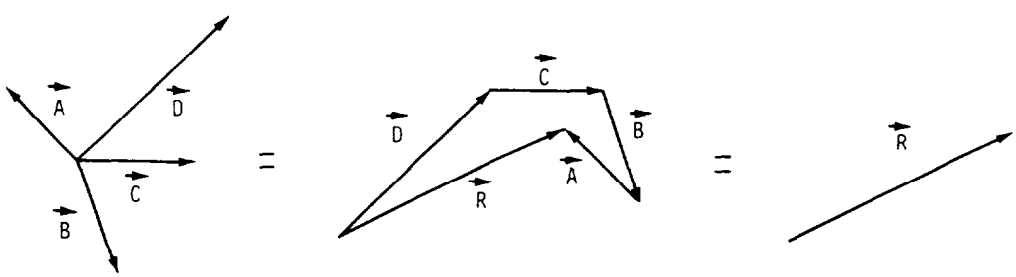

Figure 7-5.—Addition of $\vec{D} + \vec{C} + \vec{B} + \vec{A}$.

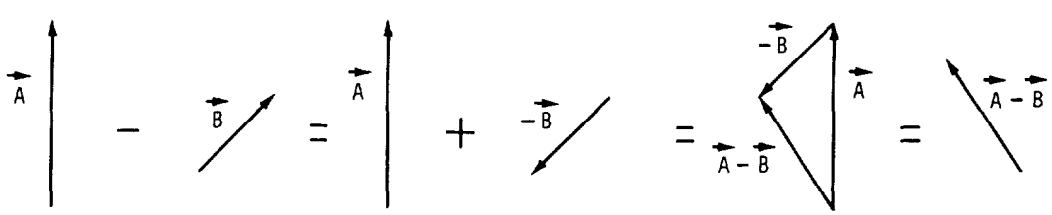

Figure 7-6.—Vector subtraction.

the negative of $\vec{B}$, denoted by $-\vec{B}$. The negative of a vector is a vector that is parallel, the same length, and points in the opposite direction. Hence, we add $-\vec{B}$ to $\vec{A}$ as previously discussed. This process may be summarized as

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

COMPONENTS OF VECTORS

The projections of a vector onto the X and Y axes of the rectangular coordinate system are called the *components* of a vector. We say that a vector is resolved into its x and y components, called the *horizontal* and *vertical components* of a vector, respectively. In figure 7-7, view A,

$$\vec{V}_x = \text{horizontal component of } \vec{V}$$

and

$$\vec{V}_y = \text{vertical component of } \vec{V}$$

Figure 7-7, view B, shows the magnitudes of $\vec{V}$, $\vec{V}_x$, and $\vec{V}_y$. Using properties of right triangles, we see that

$$\cos\theta = \frac{V_x}{V}$$

or

$$V_x = V\cos\theta$$

and

$$\sin\theta = \frac{V_y}{V}$$

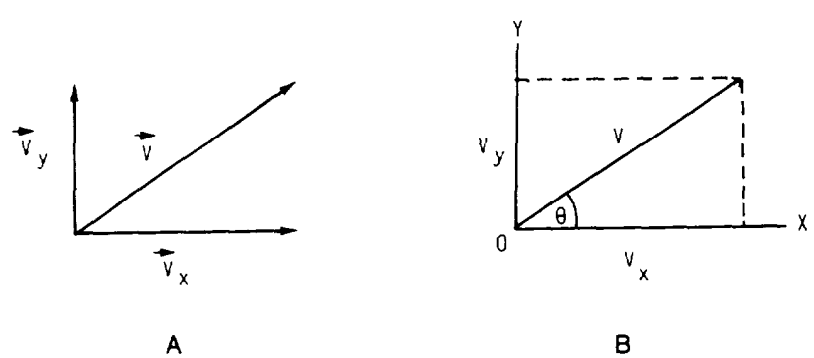

Figure 7-7.—Components of a vector.

or

$$V_y = V \sin \theta$$

where V_x, V_y, and V are the magnitudes of $\vec{V}_x$, $\vec{V}_y$, and $\vec{V}$, respectively. Notice also that (by use of the Pythagorean theorem) *the magnitude of V can be found to be*

$$V^2 = V_x^2 + V_y^2$$

or

$$V = \sqrt{V_x^2 + V_y^2}$$

The direction of $\vec{V}$ is the angle, θ, the vector makes with the horizontal. This direction can be determined by

$$\tan \theta = \frac{V_y}{V_x}$$

or

$$\theta = \arctan \frac{V_y}{V_x}$$

The direction of $\vec{V}_x$ is 0° and the direction of $\vec{V}_y$ is 90°.

EXAMPLE: Find the magnitude of the horizontal and vertical components of a vector having a magnitude of 50 pounds acting at an angle of 30° to the horizontal.

SOLUTION:

$$\begin{aligned} V_x &= V \cos \theta \\ &= 50 \cos 30° \\ &= 50\left(\frac{\sqrt{3}}{2}\right) \\ &= 25\sqrt{3} \\ &= 43.3 \text{ pounds (rounded)} \end{aligned}$$

and

$$\begin{aligned} V_y &= V \sin \theta \\ &= 50 \sin 30° \\ &= 50\left(\frac{1}{2}\right) \\ &= 25 \text{ pounds} \end{aligned}$$

EXAMPLE: Find the magnitude and direction of a vector whose horizontal and vertical components have a magnitude of 90 newtons and 60 newtons, respectively.

SOLUTION: The magnitude of the resultant vector is

$$\begin{aligned} V &= \sqrt{V_x^2 + V_y^2} \\ &= \sqrt{(90)^2 + (60)^2} \\ &= \sqrt{8{,}100 + 3{,}600} \\ &= \sqrt{11{,}700} \\ &= 108.2 \text{ newtons (rounded)} \end{aligned}$$

The direction of the resultant vector is

$$\begin{aligned} \theta &= \arctan \frac{60}{90} \\ &= \arctan 0.66667 \\ &= 33^\circ\ 41' \text{ (to the nearest minute)} \end{aligned}$$

VECTOR ADDITION BY COMPONENTS

We can add vectors that lie in the same plane by working in terms of their components. This procedure is as follows:

1. *Resolve the given vectors into their x and y components.*
2. *Add the magnitudes of the x components to give R_x (the magnitude of the x component of $\vec{R}$), and add the magnitudes of the y components to give R_y (the magnitude the y component of $\vec{R}$); that is,*

   $$R_x = A_x + B_x + C_x + \ldots$$

   and

   $$R_y = A_y + B_y + C_y + \ldots$$

3. *Find the magnitude and direction of $\vec{R}$ from R_x and R_y. The magnitude can be determined by the use of the Pythagorean theorem; that is,*

   $$R = \sqrt{R_x^2 + R_y^2}$$

   The direction of $\vec{R}$ can be found from the values of the components by trigonometry; that is,

   $$\theta = \arctan \frac{R_y}{R_x}$$

EXAMPLE: A girl walks 110 feet south, 100 feet east, 120 feet northeast, and then 90 feet northwest. What is the magnitude and direction from her starting point?

SOLUTION: For convenience we will call north the positive y direction, south the negative y direction, east the positive x direction, and west the negative x direction on a rectangular coordinate system. Refer to figure 7-8, view A, for the path the girl walks. Figure 7-8, view B, shows the magnitude and direction of each vector from the origin according to the rectangular coordinate system. Hence, the magnitudes of the components of $\vec{A}$ are

$$\begin{aligned} A_x &= A \cos \theta \\ &= 110 \cos 270° \\ &= 110(0) \\ &= 0 \text{ feet} \end{aligned}$$

and

$$\begin{aligned} A_y &= A \sin \theta \\ &= 110 \sin 270° \\ &= 110(-1) \\ &= -110 \text{ feet} \end{aligned}$$

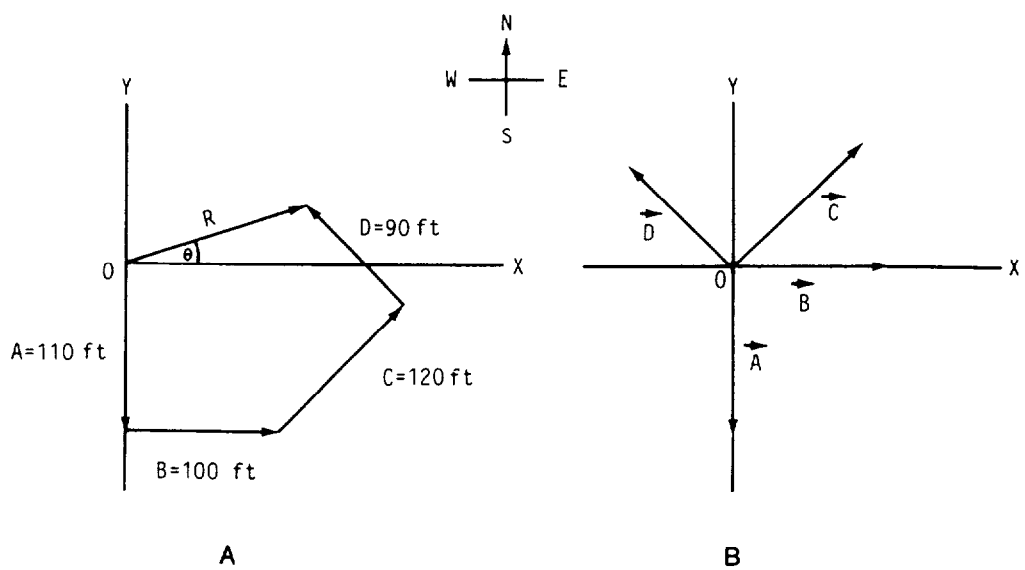

Figure 7-8.—Vector addition by components.

The magnitudes of the components of $\vec{B}$ are

$$\begin{aligned} B_x &= B \cos \theta \\ &= 100 \cos 0° \\ &= 100(1) \\ &= 100 \text{ feet} \end{aligned}$$

and

$$\begin{aligned} B_y &= B \sin \theta \\ &= 100 \sin 0° \\ &= 100(0) \\ &= 0 \text{ feet} \end{aligned}$$

The magnitudes of the components of $\vec{C}$ are

$$\begin{aligned} C_x &= C \cos \theta \\ &= 120 \cos 45° \\ &= 120\left(\frac{\sqrt{2}}{2}\right) \\ &= 60\sqrt{2} \text{ feet} \end{aligned}$$

and

$$\begin{aligned} C_y &= C \sin \theta \\ &= 120 \sin 45° \\ &= 120\left(\frac{\sqrt{2}}{2}\right) \\ &= 60\sqrt{2} \text{ feet} \end{aligned}$$

And the magnitudes of the components of $\vec{D}$ are

$$\begin{aligned} D_x &= D \cos \theta \\ &= 90 \cos 135° \\ &= -90 \cos 45° \\ &= -90\left(\frac{\sqrt{2}}{2}\right) \\ &= -45\sqrt{2} \text{ feet} \end{aligned}$$

and

$$\begin{aligned} D_y &= D \sin \theta \\ &= 90 \sin 135° \\ &= 90 \sin 45° \\ &= 90\left(\frac{\sqrt{2}}{2}\right) \\ &= 45\sqrt{2} \text{ feet} \end{aligned}$$

Now we will add the magnitudes of the x components to get R_x and add the magnitudes of the y components to get R_y:

$$\begin{aligned} R_x &= A_x + B_x + C_x + D_x \\ &= 0 + 100 + 60\sqrt{2} - 45\sqrt{2} \\ &= 121.21 \text{ feet (rounded)} \end{aligned}$$

and

$$\begin{aligned} R_y &= A_y + B_y + C_y + D_y \\ &= -110 + 0 + 60\sqrt{2} + 45\sqrt{2} \\ &= 38.49 \text{ feet (rounded)} \end{aligned}$$

Therefore, the magnitude of the resultant vector from the girl's starting point is

$$\begin{aligned} R &= \sqrt{R_x^2 + R_y^2} \\ &= \sqrt{(121.21)^2 + (38.49)^2} \\ &= \sqrt{16{,}173.34} \\ &= 127.17 \text{ feet (rounded)} \end{aligned}$$

and the direction from her starting point is

$$\begin{aligned} \theta &= \arctan \frac{R_y}{R_x} \\ &= \arctan \frac{38.49}{121.21} \\ &= \arctan 0.31755 \\ &= 17° \, 37' \text{ north of east} \end{aligned}$$

PRACTICE PROBLEMS:

Give magnitude accuracy to one decimal place and angle accuracy to the nearest minute for the following problems:

1. Find V_x and V_y of $\vec{V}$ having a magnitude of 325 pounds making an angle of 78° 20′ with the horizontal component vector.

2. Two vectors, having magnitudes of 150 newtons and 220 newtons, act at right angles to each other. Find the magnitude of their resultant vector and the angle it makes with the larger vector.

3. An airplane is heading due east with an airspeed (speed relative to the air) of 350 mph. A wind is blowing from due south at 58 mph.

   a. Find the airplane's angle of drift (the angle between its heading and its actual course).

   b. Find the ground speed (actual speed along its course).

4. Given three vectors with magnitudes and directions of 40 feet, 60°; 60 feet, 150°; and 80 feet, 225°, find the magnitude and the direction of the resultant vector.

ANSWERS:

1. V_x = 65.7 pounds

   V_y = 318.3 pounds

2. V = 266.3 newtons

   θ = 34° 17′

3. a. 9° 25′ north of east

   b. 354.8 mph

4. V = 88.9 feet

   θ = 174° 50′

FORCES

A *force* produces or prevents motion or has the tendency to do so. The effect of a force upon a body depends upon the magnitude and direction of the force. Therefore, *a force can be represented by a vector quantity*. The *resolution of a force*, then, is the separation of a single force into two or more component forces acting in given directions on the same point. Moreover, when two or more forces act on the same body, the *resultant force* is the single force whose effect upon the body is equal in magnitude and direction to the combined effects of all the forces acting on the body.

EXAMPLE: Two dogs on leashes held by a person are trying to move in directions perpendicular to each other, one pulling with a force of 64 pounds, the other with a 52-pound force. Find the magnitude of the resultant force and the angle it makes with the larger force vector.

SOLUTION: Refer to figure 7-9. If we let $\vec{F}$ be the force vector, then $\vec{F}_x$ and $\vec{F}_y$ are the two components at right angles to each other. If point A is the person holding the two dogs, $\overrightarrow{AB}$ the larger force vector, and $\overrightarrow{AD}$ the smaller force vector, then

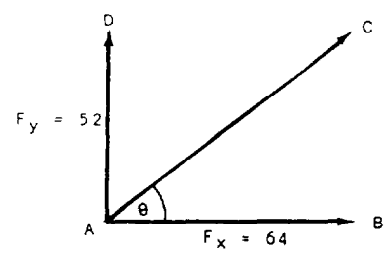

Figure 7-9.—Force vectors.

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(AB)^2 + (\mathrm{AD})^2}$$

$$= \sqrt{(64)^2 + (52)^2}$$

$$= \sqrt{6{,}800}$$

$$= 82.5 \text{ pounds (rounded)}$$

and

$$\theta = \arctan \frac{F_y}{F_x}$$

$$= \arctan \frac{52}{64}$$

$$= \arctan 0.81250$$

$$= 39^\circ\ 6'$$

EXAMPLE: An automobile weighing 3,200 pounds is parked on a driveway that makes a 17° angle with the horizontal. Find the components of the car's weight parallel and perpendicular to the driveway.

SOLUTION: The weight of an object is the gravitational force the earth exerts on it, which is a force that acts vertically downward. (See fig. 7-10.) Since $\vec{F} = \vec{W}$ is vertical and $\vec{F}_x$ is perpendicular to the driveway, then the angle between $\vec{F}$ and $\vec{F}_x$ is also $\theta = 17°$. Hence,

$$F_x = F \cos \theta$$

$$= W \cos \theta$$

$$= 3{,}200 \cos 17°$$

$$= 3{,}200(0.95630)$$

$$= 3{,}060.2 \text{ pounds (rounded)}$$

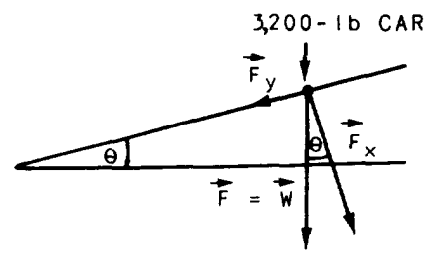

Figure 7-10.—Gravitational force.

which is the car's weight perpendicular to the driveway, and

$$F_y = F \sin \theta$$

$$= W \sin \theta$$

$$= 3{,}200 \sin 17°$$

$$= 3{,}200(0.29237)$$

$$= 935.6 \text{ pounds (rounded)}$$

which is the car's weight parallel to the driveway.

PRACTICE PROBLEMS:

1. A force of 235 pounds makes an angle of 60° 40′ with the vertical. Resolve the force into its horizontal and vertical components. Give component accuracy to one decimal place.

2. Two forces, 15 pounds at 335° and 25 pounds at 14°, act on the same point. Determine the magnitude (to one decimal place) and direction (to the nearest minute) of the resultant force.

3. A force of 53.5 pounds is acting on an object at a 25° angle to the horizontal. Find the horizontal and vertical components of the force. Give component accuracy to two decimal places.

ANSWERS:

1. $\vec{F}_x = 204.9$ pounds

   $\vec{F}_y = 115.1$ pounds

2. $F = 37.9$ pounds

   $\theta = 359° \, 34'$

3. $\vec{F}_x = 48.49$ pounds

   $\vec{F}_y = 22.61$ pounds

EQUILIBRIUM

If a body undergoes no change in its motion, it is said to be in a state of *equilibrium*. Two conditions are required for a body at rest to be in equilibrium. The body must have neither translatory (straight line) motion nor rotary (spinning) motion.

When two or more forces act together at a point, the *equilibriant force* is that single force applied at the same point which produces equilibrium. *The equilibriant force has a magnitude equal to that of the resultant of the separate forces, but it acts in the opposite direction.*

EXAMPLE: A force of 17 newtons at 123° and a force of 33 newtons at 333° act on the same point. Determine the magnitudes and directions of both the resultant and the equilibriant.

SOLUTION: For the force of 17 newtons at 123°, the magnitudes of the horizontal and vertical components are

$$A_x = 17 \cos 123°$$

$$= -17 \cos 57°$$

$$= -9.3 \text{ newtons (rounded)}$$

and

$$A_y = 17 \sin 123°$$

$$= 17 \sin 57°$$

$$= 14.3 \text{ newtons (rounded)}$$

For the force of 33 newtons at 333°, the magnitudes of the horizontal and vertical components are

$$B_x = 33 \cos 333°$$

$$= 33 \cos 27°$$

$$= 29.4 \text{ newtons (rounded)}$$

and

$$B_y = 33 \sin 333°$$

$$= -33 \sin 27°$$

$$= -15.0 \text{ newtons (rounded)}$$

Hence, the magnitudes of the horizontal and vertical components of the resultant vector are

$$F_x = A_x + B_x$$

$$= -9.3 + 29.4$$

$$= 20.1 \text{ newtons}$$

and

$$F_y = A_y + B_y$$

$$= 14.3 + -15.0$$

$$= -0.7 \text{ newtons}$$

Since the resultant and the equilibriant have equal magnitudes, then

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(20.1)^2 + (-0.7)^2}$$

$$= 20.1 \text{ newtons (rounded)}$$

The direction of the resultant is

$$\theta = \arctan \frac{F_y}{F_x}$$

$$= \arctan \frac{-0.7}{20.1}$$

$$= -2°$$

$$= 358°$$

(since our resultant force is in the fourth quadrant) and the direction of the equilibriant is

$$\theta' = \theta \pm 180°$$

$$= 358° - 180°$$

$$= 178°$$

TRANSLATIONAL EQUILIBRIUM

The first condition for equilibrium, no translatory motion, is met when no unbalanced forces act on a body. Therefore, *the sum of the forces acting on a body in any direction must be equal to the sum of the forces acting on a body in the opposite direction.*

Since the sum of all forces acting on a body must equal zero, then the sum of all the magnitudes of the horizontal components

must equal zero and the sum of all the magnitudes of the vertical components must equal zero; that is,

$$F_x = 0$$

and

$$F_y = 0$$

EXAMPLE: Find the tension (a force acting against the resistance of a body) of a weightless rope supporting a 50-pound block.

SOLUTION: Refer to figure 7-11. Since there are no horizontal components in this problem, we only need to find the magnitude of the vertical component of the tension, $\vec{T}_y$, and the magnitude of the vertical component of the weight of the block, $\vec{W}_y$, such that

$$F_y = T_y + W_y = 0$$

or

$$T_y = -W_y$$

Since

$$W_y = W \sin \theta$$

$$= 50 \sin 270°$$

$$= -50 \text{ pounds}$$

then

$$T_y = -W_y$$

$$= -(-50)$$

$$= 50 \text{ pounds}$$

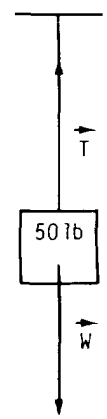

Figure 7-11.—Weight supported by a rope.

Therefore, the tension in the rope is equal to the weight being supported.

EXAMPLE: A weight of 10 newtons is supported by two cords. One cord makes an angle of 30° with the horizontal while the other makes an angle of 60° with the horizontal. Find the tension in each cord.

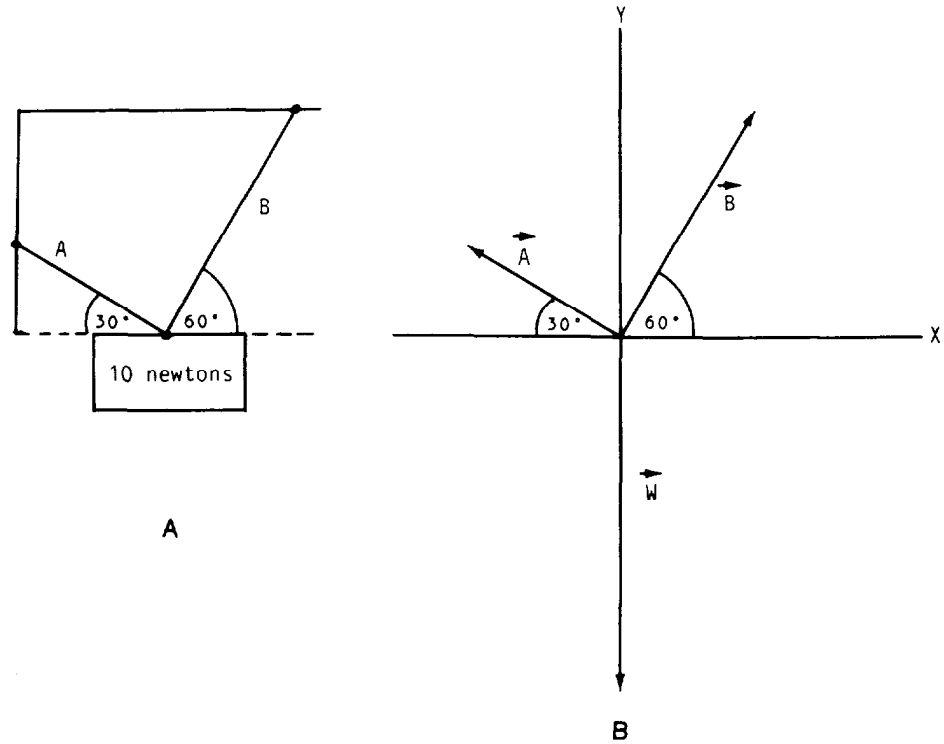

Figure 7-12.—Weight supported by two cords.

SOLUTION: Refer to figure 7-12. We begin by determining the magnitudes of the horizontal and vertical components of each vector. For tension $\vec{A}$, the reference angle (in the second quadrant) of sin θ is positive and the reference angle of cos θ is negative; so

$$A_x = -A \cos \theta$$

$$= -A \cos 30°$$

$$= -\frac{\sqrt{3}}{2}A \text{ newtons}$$

and

$$A_y = A \sin \theta$$

$$= A \sin 30°$$

$$= \frac{1}{2}A \text{ newtons}$$

For tension $\vec{B}$, the reference angles of $\sin\theta$ and $\cos\theta$ are both positive in quadrant I; so

$$\begin{aligned} B_x &= B\cos\theta \\ &= B\cos 60^\circ \\ &= \frac{1}{2}B \text{ newtons} \end{aligned}$$

and

$$\begin{aligned} B_y &= B\sin\theta \\ &= B\sin 60^\circ \\ &= \frac{\sqrt{3}}{2}B \text{ newtons} \end{aligned}$$

For the weight of 10 newtons,

$$\begin{aligned} W_x &= W\cos\theta \\ &= 10\cos 270^\circ \\ &= 0 \text{ newtons} \end{aligned}$$

and

$$\begin{aligned} W_y &= W\sin\theta \\ &= 10\sin 270^\circ \\ &= -10 \text{ newtons} \end{aligned}$$

Now, since the sum of the magnitudes of the horizontal components must equal zero and the sum of the magnitudes of the vertical components must equal zero, then

$$A_x + B_x + W_x = 0$$

or

$$\begin{aligned} -\frac{\sqrt{3}}{2}A + \frac{1}{2}B &= 0 \\ -\frac{\sqrt{3}}{2}A &= -\frac{1}{2}B \\ \sqrt{3}A &= B \end{aligned}$$

and

$$A_y + B_y + W_y = 0$$

or

$$\frac{1}{2}A + \frac{\sqrt{3}}{2}B + -10 = 0$$

$$\frac{1}{2}A + \frac{\sqrt{3}}{2}B = 10$$

Substituting $\sqrt{3}A$ for B in the last equation, we obtain

$$\frac{1}{2}A + \frac{\sqrt{3}}{2}(\sqrt{3}A) = 10$$

$$\frac{1}{2}A + \frac{3}{2}A = 10$$

$$2A = 10$$

$$A = 5 \text{ newtons}$$

and

$$B = \sqrt{3}A$$

$$= \sqrt{3}(5)$$

$$= 8.7 \text{ newtons (rounded)}$$

ROTATIONAL EQUILIBRIUM

The second condition for equilibrium, no rotary motion, is met when the sum of the torques acting upon a body about a point equals zero; that is,

$$\tau_R = \tau_1 + \tau_2 + \tau_3 + \ldots = 0$$

Hence, *the sum of all the clockwise torques equals the sum of all the counterclockwise torques about an axis of rotation. Torque* is the product of the magnitude of a force, F, and the length of its torque or lever arm, L, where L is measured perpendicular to the line of action of the force. Hence,

$$\tau_R = F_1L_1 + F_2L_2 + F_3L_3 + \ldots = 0$$

If a force tends to produce a counterclockwise rotation about an axis, the torque will be considered positive. If a force tends to produce a clockwise rotation about an axis, the torque will be considered negative.

EXAMPLE: A person exerts a 15-pound force at the end of an 8-inch wrench. (See fig. 7-13.) If this force makes an angle of 45° with the handle, what is the torque produced on the nut?

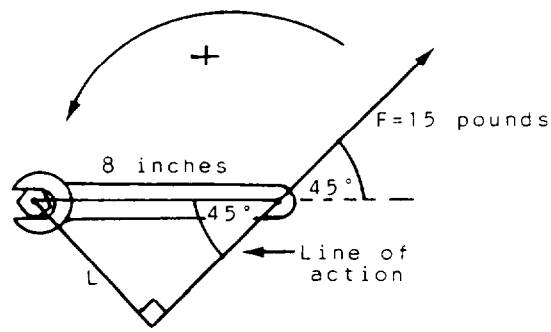

Figure 7-13.—Torque.

SOLUTION: First we need to find the length of the torque arm. Since L is measured perpendicular to the line of action, then to solve for L, we will use

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

or

$$\sin 45° = \frac{L}{8}$$

$$L = 8 \sin 45°$$

$$= 8\left(\frac{\sqrt{2}}{2}\right)$$

$$= 4\sqrt{2} \text{ inches}$$

Since the force tends to produce a counterclockwise rotation about the axis, then the torque produced on the nut is positive and

$$\tau = FL$$

$$= 15(4\sqrt{2})$$

$$= 60\sqrt{2}$$

$$= 84.9 \text{ pound} \cdot \text{inches (rounded)}$$

EXAMPLE: A rod 12 meters long has weights of 5 newtons and 15 newtons at its ends. (Assume that the weight of the rod is negligible.) At what point should the rod be picked up if it is to have no tendency to rotate (where is the balance point of the rod)?

SOLUTION: Refer to figure 7-14. First, compute the torques about the unknown balance point. If x is the distance of the 15-newton weight from this point, then the 5-newton weight is $(12 - x)$ meters from the point on the other side. Note that x is the length of the torque arm of the 15-newton weight and $(12 - x)$ is the length of the torque arm of the 5-newton weight. Since the 15-newton weight tends to produce a counterclockwise rotation about the balance point of the rod, then the torque will be considered positive. Likewise, since the 5-newton weight tends to produce a clockwise rotation about the balance point, then the torque will be considered negative. Hence,

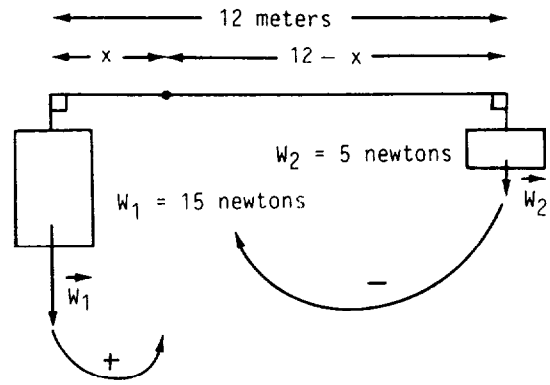

Figure 7-14.—Balanced rod.

$$\tau_1 = F_1L_1$$
$$= 15(x)$$
$$= 15x$$

and

$$\tau_2 = F_2L_2$$
$$= -5(12 - x)$$
$$= -60 + 5x$$

Since no rotary motion occurs when

$$\tau_R = \tau_1 + \tau_2 = 0$$

then

$$F_1L_1 + F_2L_2 = 0$$
$$15x - 60 + 5x = 0$$
$$20x = 60$$
$$x = 3 \text{ meters}$$

Therefore, when the rod is picked up 3 meters from the 15-newton weight end, the two weights exert opposite torques of the same magnitude $[15x = 45 = 5(12 - x)]$ about this point, where the rod is balanced.

PRACTICE PROBLEMS:

Give magnitude accuracy to one decimal place and angle accuracy to the nearest minute for the following:

1. Find the magnitudes and directions of the resultant and equilibriant forces of a force of 21 newtons due north and a second force of 32 newtons southeast.

2. A 20-pound ball is suspended by a rope, A, attached to a wall. Rope A is pulled away from the wall by a horizontal rope, B, and is held so that rope A forms an angle of 30° with the vertical wall. Find the tensions in ropes A and B.

3. A 150-newton force is applied to a pole 6 meters above its base at an angle of 45° above the horizontal. Find the torque about the base of the pole.

4. A uniform horizontal bar is 550 millimeters long and is of negligible weight. A 32-newton weight is hung from the left end of the bar, and a 70-newton weight is hung from the right end. Where should a single upward support be positioned to balance the system?

ANSWERS:

1. $R = 22.7$ newtons

   $E = 22.7$ newtons

   $\theta_R = 355° \, 57'$

   $\theta_E = 175° \, 57'$

2. $A = 23.1$ pounds

   $B = 11.5$ pounds

3. 636.4 newton · meters

4. 172.5 millimeters from the right end

SUMMARY

The following are the major topics covered in this chapter:

1. **Definitions:**

   A *scalar quantity* is one that has magnitude only.

   A *vector quantity* is one that has both magnitude and direction.

2. **Vectors:**

   Two vectors are said to be equal if they are of the same length, are parallel, and point in the same direction.

   If two vectors have the same length, are parallel, but point in opposite directions, they are said to be negatives of each other.

3. **Resultant vectors:** To find the resultant vector of any number of vectors, place the initial end of each vector to the terminal end of the previous vector. Be sure the new vector position is parallel, the same length, and in the same direction as the old vector position. Draw a vector from the initial end of the first vector to the terminal end of the last vector. This newly formed vector is the *resultant vector*.

4. **Vector addition:**

$$\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

   where $\vec{A}$ and $\vec{B}$ are added and $\vec{R}$ is their resultant vector.

5. **Vector subtraction:**

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

   where $\vec{B}$ is subtracted from $\vec{A}$, which is the same as adding the negative of $\vec{B}$ to $\vec{A}$.

6. **Components of vectors:** The projections of a vector onto the X and Y axes of the rectangular coordinate system are called the *components* of a vector. The *horizontal component* of $\vec{V}$ is $\vec{V}_x$ and the *vertical component* of $\vec{V}$ is $\vec{V}_y$.

7. **Magnitudes of vectors:** The magnitudes of $\vec{V}_x$, $\vec{V}_y$, and $\vec{V}$, respectively, are

$$V_x = V \cos \theta$$

$$V_y = V \sin \theta$$

$$V = \sqrt{V_x^2 + V_y^2}$$

8. **Directions of vectors:** The direction of $\vec{V}$ is the angle, θ, the vector makes with the horizontal.

$$\theta = \arctan \frac{V_y}{V_x}$$

The direction of $\vec{V}_x$ is 0° and the direction of $\vec{V}_y$ is 90°.

9. **Vector addition by components:**

   1. Resolve the given vectors into their x and y components.

   2. Add the magnitudes of the x components to give R_x (the magnitude of the x component of $\vec{R}$), and add the magnitudes of the y components to give R_y (the magnitude of the y component of $\vec{R}$); that is,

$$R_x = A_x + B_x + C_x + \ldots$$

   and

$$R_y = A_y + B_y + C_y + \ldots$$

   3. Find the magnitude and direction of $\vec{R}$ from R_x and R_y. The magnitude can be determined by the use of the Pythagorean theorem; that is,

$$R = \sqrt{R_x^2 + R_y^2}$$

   The direction of $\vec{R}$ can be found from the values of the components by trigonometry; that is,

$$\theta = \arctan \frac{R_y}{R_x}$$

10. **Force:** A *force* produces or prevents motion or has the tendency to do so.

    A force can be represented by a vector quantity.

The *resolution of a force* is the separation of a single force into two or more component forces acting in given directions on the same point.

When two or more forces act on the same body, the *resultant force* is the single force whose effect upon the body is equal in magnitude and direction to the combined effects of all the forces acting on the body.

11. **Equilibrium:** If a body undergoes no change in its motion, it is said to be in a state of *equilibrium*.

    For a body at rest to be in equilibrium, it must have neither translatory motion nor rotary motion.

12. **Equilibriant force:** When two or more forces act together at a point, the *equilibriant force* is that single force applied at the same point which produces equilibrium.

    The equilibriant force has a magnitude equal to that of the resultant of the separate forces, but it acts in the opposite direction.

13. **Translational equilibrium:** The sum of the forces acting on a body in any direction must be equal to the sum of the forces acting on a body in the opposite direction, such that

$$F_x = 0$$

and

$$F_y = 0$$

14. **Rotational equilibrium:** The sum of all the clockwise torques equals the sum of all the counterclockwise torques about an axis of rotation, such that,

$$\tau_R = \tau_1 + \tau_2 + \tau_3 + \ldots = F_1L_1 + F_2L_2 + F_3L_3 + \ldots = 0$$

Torque is the product of the magnitude of force, F, and the length of its torque or lever arm, L, where L is measured perpendicular to the line of action of the force.

If a force tends to produce a counterclockwise rotation about an axis, the torque will be considered positive. If a force tends to produce a clockwise rotation about an axis, the torque will be considered negative.

ADDITIONAL PRACTICE PROBLEMS

Give magnitude accuracy to one decimal place and angle accuracy to the nearest minute for the following:

1. An airplane is heading due south at 220 mph and the wind is blowing from due east at 65 mph. Find the airplane's angle of drift and its ground speed.

2. An airplane flies 100 miles west from city A to city B, then 100 miles north from city B to city C, and finally 50 miles southeast to city D. How far is it from city A to city D?

3. Three forces act simultaneously on a point. One force is 15 newtons at 0°; the second is 20 newtons at 210°; and the third is 30 newtons at 60°. Determine the magnitude and direction of the resultant force.

4. A force of 11 pounds at 111° acts on a point. A second force of 22 pounds at 222° and a third force of 33 pounds at 333° also act on the same point. Determine the magnitude and direction of the equilibriant force.

5. A 100-pound tightrope walker stands at the center of a rope that is 200 feet in length. If the rope sags 20 feet at the center, find the tension in each side of the rope.

6. A 40-pound child and a 60-pound child sit at opposite ends of a 12-foot seesaw pivoted at its center. Where should a third child who weighs 50 pounds sit in order to balance the seesaw?

ANSWERS TO ADDITIONAL PRACTICE PROBLEMS

1. $\theta = 16° 28'$ west of south

   $S = 229.4$ mph

2. 91.4 miles

3. $F = 20.4$ newtons

   $\theta = 51° 34'$

4. $E = 21.5$ pounds

   $\theta = 115° 22'$

5. 255 pounds

6. 2.4 feet from the pivot on the same side as the 40-pound child