CHAPTER 14

DEPENDENCE, FUNCTIONS, AND FORMULAS

In chapter 13 of this course, use is made of several formulas, such as A = LW, E = IR, etc. It is the purpose of this chapter to explain the function and dependency relationships which make formulas so useful.

DEPENDENCE AND FUNCTIONS

Dependence may be defined as any relationship between two variables which allows the prediction of change in one of them as a result of change in the other. For example, the cost of 200 bolts depends upon the price per hundred. If C represents cost and p represents the price of 100 bolts, then the cost of 200 bolts may be expressed as follows:

$$C = 2p$$

In the example just given, C is called the DEPENDENT VARIABLE because its value depends upon the changing values of p. The INDEPENDENT VARIABLE is p. It is standard practice to isolate the dependent variable on the left side of an equation, as in the example.

Consider the formula for the area of a rectangle, A = LW. Here we have two independent variables, L and W.

Figure 14-1 (A) shows what happens if we double the length. Figure 14-1 (B) shows the result of doubling the width. Figure 14-1 (C) shows the effect of doubling both length and width. Notice that when the length or width alone is doubled the area is doubled, but when both length and width are doubled the area is four times as great.

In any equation showing a dependency relationship, the dependent variable is said to be a FUNCTION of the independent variable. Another use of the term "function" in describing an equation such as C = 2p is to refer to the whole expression as "the function C = 2p." This terminology is especially useful when the right-hand expression has several terms. For example, consider the equation $y = 2x^2 + 3x - 4$. Mathematicians frequently use a shorthand notation and rewrite the equation as y = f(x). The expression f(x) is understood to mean "a function of x" and reference to the function by calling it f(x) saves the space and time that would otherwise be required to write out all three terms.

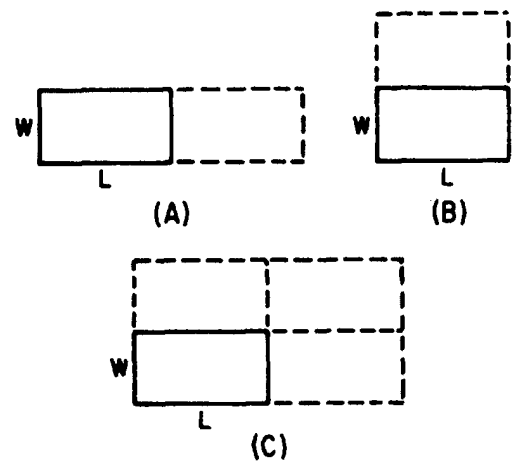


Figure 14-1.—Changes in the area of a rectangle resulting from changes in length and width.

Practice problems. Answer the following questions concerning the function $r = \frac{d}{t}$.

1. When t increases and d remains the same, does r increase, decrease, or remain the same?

2. When d increases and t remains the same, does r increase, decrease, or remain the same?

3. When t decreases and d remains the same, does r increase, decrease, or remain the same?

4. When d decreases and t remains the same, does r increase, decrease, or remain the same?

5. When d is doubled and t remains the same, is r doubled or halved?

6. When t is doubled and d remains the same, is r doubled or halved?

Answers:

1. Decreases.
2. Increases.
3. Increases.
4. Decreases.
5. Doubled.
6. Halved.

FORMULAS

One of the most common uses of algebra is in the solution of formulas. Formulas have a wide and varied use throughout the Navy. It is important to know how formulas are derived, how to translate them into words, how to make them from word statements, and how to use them to solve problems.

A formula is a general fact, rule, or principle expressed in algebraic symbols. It is a shorthand expression of a rule in which letters and signs of operation take the place of words. The formula always indicates the mathematical operations involved. For example, the formula $P = 2L + 2W$ indicates that the perimeter (sum of the lengths of the sides) of a rectangle is equal to twice its length plus twice its width. (See fig. 14-2.)

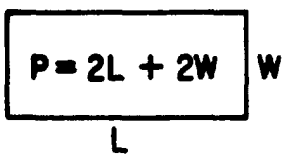


Figure 14-2.—Perimeter of a rectangle.

A formula obtained by logical or mathematical reasoning is called a mathematical formula. A formula whose reliability is based on a limited number of observations, or on immediate experience, and not necessarily on established theories or laws is called an EMPIRICAL formula. Empirical formulas are found frequently in engineering and physical sciences. They sometimes are valid for only a limited number of values.

SUBJECT OF A FORMULA

Usually a formula is taken almost directly from the verbal rule or law. For instance, the perimeter of a rectangle is equal to twice the length plus twice the width. Where possible, letters are used as symbols for the words. Thus, $P = 2L + 2W$. A simple formula such as this is like a declarative sentence. The left half is the SUBJECT and all the rest is the predicate. The subject is P. It corresponds to the part of the verbal rule that reads "the perimeter of a rectangle." This subject is usually a single letter followed by the equality sign.

All formulas are equations, but not all equations are formulas. Some distinctions between a formula and an ordinary equation are worthy of note. The equation may not have a subject, while the formula typically does. In the formula, the unknown quantity stands alone in the left-hand member. No computation is performed upon it, and it does not appear more than once. In the equation, on the other hand, the unknown quantity may appear once or more in either or both members, and computation may be performed with it or on it. We evaluate a formula by substituting for the literal numbers in the right member. An equation is solved by computation in either or both members until all that remains is an unknown in one member and a known quantity in the other. The solution of an equation usually requires a knowledge of algebraic principles, while the evaluation of a formula may ordinarily be accomplished with only a knowledge of arithmetic.

SYMBOLS

Letters that represent words have been standardized in many cases so that certain formulas may be written the same in various texts and reference books. However, to avoid any misunderstanding a short explanation often accompanies formulas as follows:

$$A = hw,$$

where

A = area in square units

h = height

w = width

Subscripts and Primes

In a formula in which two or more of the same kind of letters are being compared, it is desirable to make a distinction between them. In electronics, for example, a distinction between resistances may be indicated by R_a and R_b or R_1 and R_2. These small numbers or

letters written to the right and below the R's are called subscripts. Those shown here are read: R sub a, R sub b, R sub one, and R sub two. Primes are also used in the same manner to distinguish between quantities of the same kind. Primes are written to the right and above the letters, as in S', S'', and S'''. They are read: S prime, S double prime, and S triple prime.

CHANGING THE SUBJECT OF A FORMULA

If values are given for all but one of its variables, a formula can be solved to obtain the value of that variable. The first step is usually the rearranging of the formula so that the unknown value is the subject—that is, a new formula is derived from the original. For example, the formula for linear motion—distance equals rate times time—is usually written

$$d = rt$$

Suppose that instead of the distance we wish to know the rate, r, or the time, t. We simply change the subject of the formula by the algebraic means developed in earlier chapters. Thus, in solving the formula for r, we divide both sides by t, with the following result:

$$\frac{d}{t} = \frac{rt}{t}$$

$$\frac{d}{t} = r, \text{ or } r = \frac{d}{t}$$

In words, this formula states that rate equals distance divided by time. Likewise, in solving for t, we have the following :

$$\frac{d}{r} = \frac{rt}{r}$$

$$\frac{d}{r} = t, \text{ or } t = \frac{d}{r}$$

In words, this formula states that time equals distance divided by rate.

We have in effect two new formulas, the subject of one being rate and the subject of the other being time. They are related to the original formula because they were derived from it, but they are different in that they have different subjects.

Practice problems. Derive new formulas from the following expressions with subjects as indicated:

1. $A = \frac{1}{2}bh$, subject h
2. $P = 2L + 2W$, subject L
3. $i = prt$, subject r
4. $p = br$, subject b
5. $E = IR$, subject I
6. The modern formula for converting Fahrenheit temperatures to Celsius (centigrade) is $C = (F + 40)\left(\frac{5}{9}\right) - 40$. Express the formula for converting Celsius (centigrade) temperatures to Fahrenheit.

Answers:

1. $h = \frac{2A}{b}$
2. $L = \frac{P - 2W}{2}$
3. $r = \frac{i}{pt}$
4. $b = \frac{p}{r}$
5. $I = \frac{E}{R}$
6. $F = (C + 40)\frac{9}{5} - 40$

EVALUATING FORMULAS

The first step in finding the value of the unknown variable of a formula is usually the derivation of a formula that has the unknown as its subject. Once this is accomplished, the evaluation of a formula consists of nothing more than substituting numerical values for the letters representing known quantities and performing the indicated operations.

For example, suppose we wish to find the time required to fly 1,250 nautical miles at the rate of 250 knots. The formula is d = rt. We can change the subject by dividing both sides of the equation by r, as follows:

$$\frac{d}{r} = \frac{rt}{r}$$

$$\frac{d}{r} = t$$

$$t = \frac{1250}{250} = 5 \text{ hr}$$

Formulas can be solved for an unknown by substituting directly in the original formula

even though that unknown is not the subject. Generally, however, it is simpler to first make the unknown the subject.

Formulas vary widely, from the simple type such as we have been considering to some that are very complex. All formulas have certain characteristics in common. There is always a subject, the quantity whose value is sought as a final answer. This subject usually stands alone, being placed equal to at least one and possibly several literal numbers, which are combined according to certain indicated operations. The formula can always be evaluated for a specific case when numerical values are known for all these literal quantities.

Evaluating formulas may be facilitated by developing a routine order of doing the work. If someone else can read the work and clearly understand what has been done, the work is in good order. The original formula should be written first, then the derived formula that is going to be used in solving the problem, and finally the actual substitutions. The indicated operations may then be carried out. Care should be taken to label answers with correct units; that is, miles per hour, foot-pounds, square feet, etc.

Practice problems.

1. $E = IR$. Solve for R in ohms if E is 110 volts and I is 5 amperes.

2. $d = rt$. Solve for t in hours if d is 840 nautical miles and r is 25 knots.

3. $F = (C + 40)\left(\frac{9}{5}\right) - 40$. Solve for C if F is 32°.

Answers:

1. 22 ohms 2. 33.6 hr 3. 0°

DEVELOPING FORMULAS

Developing a formula from a verbal statement is nothing more than reducing the statement to a shorthand form and showing the mathematical relationships between the elements of the statement.

For example, suppose that we wish to develop a formula showing the distance, D, traveled at the rate of 20 knots for t hours. If the distance traveled in 1 hr is 20 nautical miles, then the distance traveled in t hours is 20t. Therefore, the formula is

$$D = 20t$$

Practice problems.

1. Write a formula for the cost, C of p pounds of sugar at 15 cents per pound.

2. Write the formula for the cost, C, of one article when the total cost, T, of n similar articles is known.

3. Write a formula for the number of days, d, in w weeks.

4. Write a formula for the number of ounces, n, in p pounds.

Answers:

1. $C = 15p$ 3. $d = 7w$
2. $C = T/n$ 4. $n = 16p$

Developing Formulas from Tables

In technical work, instrument readings and other data are often recorded in a tabular arrangement. By careful observation of such tables of data, it is frequently possible to find values that are related in a definite pattern. The table can thus be used in developing a formula showing the relationship between the related quantities.

For example, table 14-1 shows the results of time trials on a ship, with the data rounded to the nearest whole hour and the nearest whole mile.

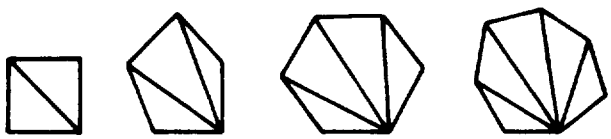
Table 14-1.—Time trials.

Nautical miles (d)	20	40	60	80	100
Hours (t)	1	2	3	4	5

By inspection of the table, it soon becomes clear that the number of miles traveled is always 20 times the corresponding number of hours. Therefore the formula developed from this table is as follows:

$$d = 20t$$

A second example of the derivation of a formula from a table is shown in figure 14-3. Figure 14-3 (A) shows several polygons (many-sided plane figures), each with one or more diagonals. A diagonal is a straight line joining one vertex (point where two sides meet) with another.



(A)

DIAGONALS (d)	1	2	3	4
SIDES (n)	4	5	6	7

(B)

Figure 14-3.—Diagonals of plane figures.

The table in figure 14-3 (B) compares the number of sides of each polygon with the number of diagonals that can be drawn from any one vertex. Using this table, we make a formula for the number, d, of diagonals that can be drawn from one vertex of a polygon of n sides. In the table we note that the number of diagonals is always 3 less than the number of sides. Therefore the formula is $d = n - 3$.

Practice problems. Complete the following tables and write formulas to show the relationship between the numbers.

1.

L	2	5	8	11	14	17	20
P	12	30	48	66	84		

2.

a	0	1	2	3	4	5	6	7
b	4	5	6	7				

3.

x	0	1	2	3	4	5	6
y	0	3	6	9	12		

4.

n	1	2	3	4	5	6	7
s	3	6	11	18	27	38	

Answers:

1. $P = 6L$
2. $b = a + 4$
3. $y = 3x$
4. $s = n^2 + 2$

TRANSLATING FORMULAS

Thus far, we have been concerned primarily with reducing verbal rules or statements to formula form. It is also necessary to be able to do the reverse, and translate a formula into words. Technical publications frequently take advantage of the fact that it is more convenient to write formulas than longhand rules. Understanding is hampered if we are not able to translate these formulas into words. As an example of translation, we may translate the formula $V = lwh$ into words, with the literal factors representing words as follows:

V = volume of a rectangular solid
l = length
w = width
h = height

This produces the following translation: The volume of a rectangular solid equals the length times the width times the height.

As a second example, we translate the algebraic expression $2\sqrt{x} - 4$ into words as follows: Twice the square root of a certain number, minus 4.

Practice problems. Translate each of the following expressions into words.

1. $PV = k$, where P represents pressure of a gas and V represents volume. (Assume constant temperature.)

2. $x = y + 4$, where x and y are numbers.

3. $A = LW$, where A is the area of a rectangle, L is its length, and W is its width.

4. $d = rt$, where d is distance, r is rate, and t is time.

Answers:

1. The pressure of a gas multiplied by its volume is constant, if the temperature is constant.

2. A certain number, x, is equal to the sum of another number, y, and 4.

3. The area of a rectangle is equal to the product of its length times its width.

4. Distance is equal to rate multiplied by time.

GRAPHING FORMULAS

We have seen that the formula is an equation. Since all formulas are equations they may be graphed. Graphs of formulas have wide use in the Navy in such fields as electronics and engineering. In practical applications it is often convenient to derive information from graphs of formulas rather than from formulas directly.

As an example, suppose that a fuel costs 30 cents per gallon. The formula for the cost in dollars of n gallons is

$$C = 0.30n$$

We see that this is a linear equation, the resulting curve of which passes through the origin (no constant term). Since we are interested only in positive values, we can eliminate three quadrants of the graph and use only the first quadrant. We already know one point on the graph is (0,0). We need plot only one other point to graph the formula. The result is shown in figure 14-4.

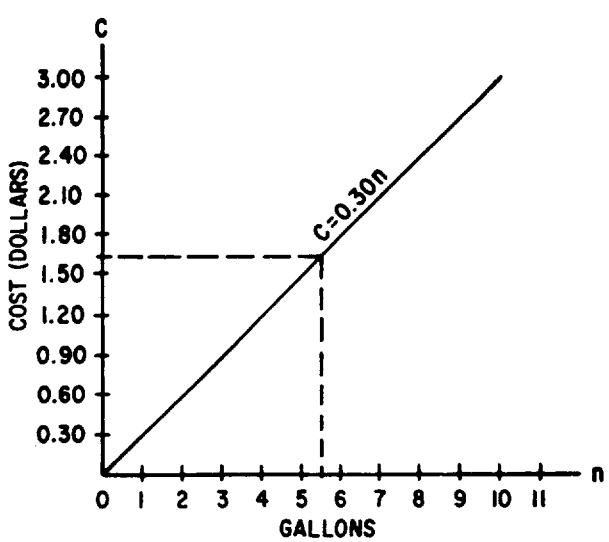


Figure 14-4.—Graph for the formula C = 0.30n.

We may read the cost directly from the graph when the number of gallons is known, or the number of gallons when the cost is known. For instance, if 5-1/2 gal are sold, find 5-1/2 on the gallons scale and follow the vertical line from that point to the point where it intersects the graph of the formula. From this point, follow the horizontal line to the cost scale. The horizontal line intersects the cost scale at 1.65. Therefore the cost of 5-1/2 gal is $1.65.

Likewise, to answer the question, "How many gallons may be bought for $1.27," we would enlarge the graph enough to estimate to the exact cent. Then we would follow a horizontal line from 1.27 on the cost scale to the formula graph and follow a vertical line from that point to the gallons scale. Thus, 4-1/4 gal may be bought for $1.27.

Plotting two formulas on the same graph may help to solve certain kinds of problems. For example, suppose that two ships leave port at the same time. One averages 10 knots and the other averages 15 knots. How far has each traveled at the end of 3 hr and at the end of 5 hr? A graph to relate the two ships' movements at any time can be made as follows: Let the vertical scale be in nautical miles and the

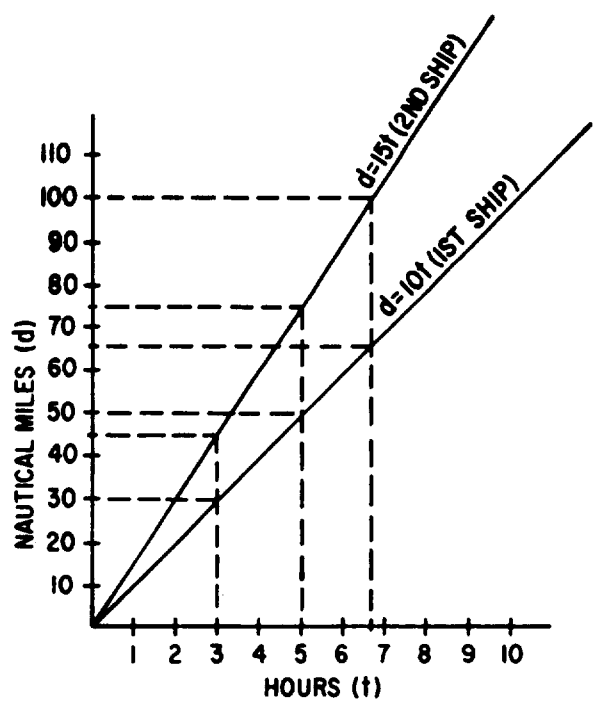


Figure 14-5.—Graph of the formulas d = 10t and d = 15t.

horizontal scale be in hours. The formula for the first ship's distance related to time is

$$d = 10t$$

The formula for the second ship's distance related to time is

$$d = 15t$$

We see that these formulas are linear and their curves pass through the origin. They are graphed in figure 14-5.

With this graph we can now answer the questions originally posed, at a glance. Thus in 3 hr the first ship traveled 30 mi and the second traveled 45 mi. In 5 hr the first ship traveled 50 mi and the second traveled 75 mi.

We could also answer such questions as: When the second ship has traveled 100 mi, how far has the other traveled? We first find the point on the graph of $d = 15t$ where the ship has traveled 100 mi. We then follow the vertical line from that point to the point where it intersects the graph of the other formula. From the point of intersection we follow a horizontal line to the distance axis and see that the first ship has traveled about 67 mi when the second has traveled 100 mi.

The foregoing examples serves to illustrate the wide variety of applications in which graphs of formulas are useful.